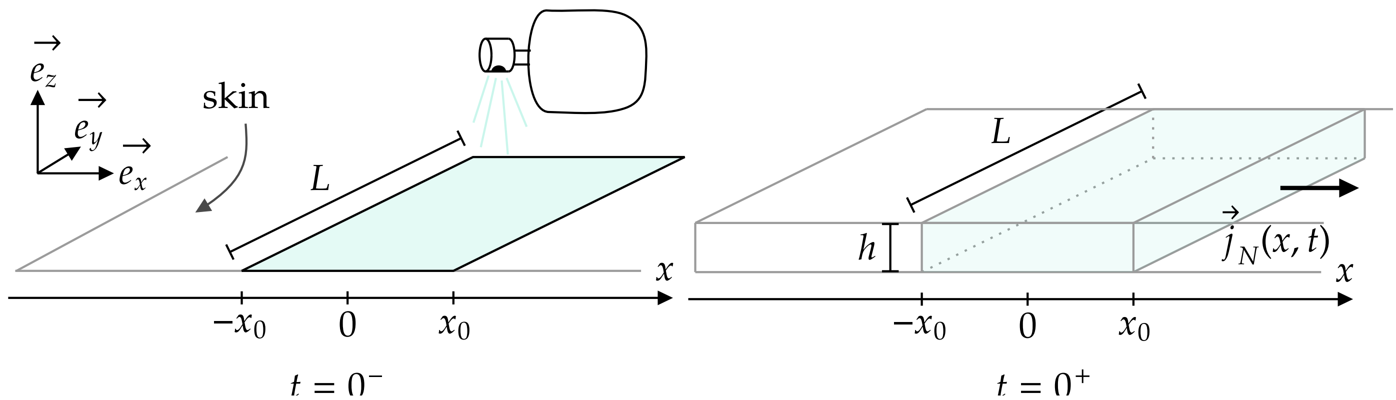
Duration : 2 h 30. The use of a calculator or calculating device is forbidden. Any affirmation must be justified.

I - Diffusion of a perfume

(33% of the points)

After initially applying perfume locally on the skin in a surface $2x_0 \times L$, we model here 1D diffusion of perfume molecules in air. Let $n(x,t)$ be the number of perfume molecules per volume of air. Due to slow evaporation the deposit of perfume liberates perfume molecules within the air on top of the $2x_0 \times L$ surface. We note α the constant number of perfume molecules added per unit of volume and per unit of time within the air layer of $-x_0 \geq x \leq x_0$.

We note D the diffusion coefficient of perfume in air, and $\vec{j}_N(x,t)$ the diffusion flux density. We assume that the plane (O, $\vec{e_y}$,$\vec{e_z}$) is Π^+, that is plane of symmetry, for n and for $\vec{j}_N$.



Q1. Define Fick's law, then justify that $\overrightarrow{j_N}(x,t) = j_N(x,t)\vec{e_x}$ with $j_N(x,t) = \overrightarrow{j_N}(x,t).\vec{e_x}$.

Q2. Establish rigorously the material balance for a layer of air between x and $x + \mathrm{d}x$ between t and $t + \mathrm{d}t$, first within $x \in [-x_0, x_0]$, then for $x \in \mathbb{R}\backslash[-x_0, x_0]$.

We now study the diffusion in steady-state.

Q3. Establish the expression of $n(x)$, with 4 unknowns that we leave undetermined for now.

We note $n(0) = n_0$, and in $x = \pm x_0$ there is no membrane that could constrain molecule movement.

Q4. State the boundary conditions in $x = 0$ and $x = x_0$, to demonstrate rigorously the following :

$$\begin{cases} \text{if } x \in [0, x_0] & n(x) = n_0 - \frac{\alpha}{2D}x^2 \\ \text{else if } x \geq x_0 & n(x) = n_0 + \frac{\alpha}{2D}x_0^2 - \frac{\alpha x_0}{D}x \end{cases}$$

A typical bottle of perfume contains 50 mL of liquid perfume of molar mass $M \simeq 100$ g.mol^{-1} and volumetric mass $\mu \simeq 10^3$ kg.m^{-3}. Such a bottle lasts about 6 months, for 2 sprays a day, each spray lasting around 5 hours before completely evaporating.

Q5. Establish an order of magnitude for α using the given data.

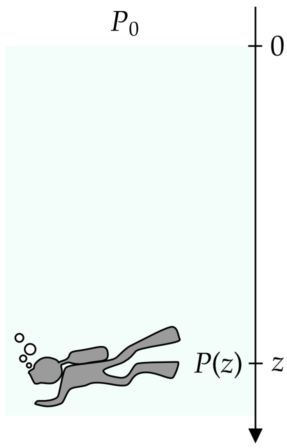
The diffusion coefficient of perfume within air is $D \simeq 3 \times 10^{-5}$ m^2.s^{-1}.

Q6. Estimate the time Δt for perfume molecules to diffuse over 1 meter of air. Relate your result to everyday life observations, and to other physical phenomenons.

II - Scuba diving accident : growth of gas bubbles

(67% of the points)

The 3 subpart of this problem are independent.



When scuba diving, the body is exposed to increasing hydrostatic pressure as depth increases. At higher pressures, more gases dissolve within the living tissues. If a diver then reaches the surface very quickly, gas trapped within the tissues does not have time to diffuse back to the blood and lungs, and instead grows into gas bubbles that can become mortal.
Throughout this study, we suppose that at equilibrium the concentration $c_{\mathrm{N_2},\,eq}$ (in mol/L) of dissolved $\mathrm{N_2}$ within a living tissue is proportional to the **partial** pressure $P_{\mathrm{N_2}}$ in $\mathrm{N_2}$ surrounding this tissue according to Henry's law : $c_{\mathrm{N_2},\,eq} = H \times P_{\mathrm{N_2}}$ with $H = 6 \times 10^{-4}$ mol.L^{-1}.bar^{-1}.

II.1 First estimation of the danger

Q7. Recall the approximate value of the molar fraction of N_2 in air, then deduce the approximate value of $c_{\mathrm{N_2},\,eq}(z=0)$ for atmospheric pressure $P_0 = P(z=0)$.

Q8. Recall without any demonstration the hydrostatic pressure profile $P(z)$ within water. Determine the approximate value of $c_{\mathrm{N_2},\,eq}(z_0)$ with $z_0 = 30$ m.

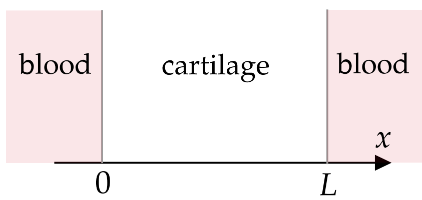
We imagine that the diver, initially at equilibrium at z_0=30 m of depth, suddenly emerges at $z = 0$. The total volume of blood of a human being is about $V = 5$ L.

Q9. Determine the amount (in moles) of $\mathrm{N_2}$ gas that appears within the diver's blood if it instantaneously reaches its new equilibrium. Using ideal gas law, convert this amount of N_2 gas into a volume of gas at atmospheric pressure, is this volume enough to obstruct a blood vessel ?

II.2 Avoid the accident : the slow diffusion of dinitrogen in living tissues

Usually gas bubbles do not emerge in the blood, which circulates very often through the lungs and thus adapts quickly its concentration with the pressure $P(z)$ within the lungs. Therefore we suppose that in blood for each depth z the concentration in $\mathrm{N_2}$ is the one at equilibrium stated by Henry's law : $c_{\mathrm{N_2},\,eq} = 5 \times 10^{-4}$ mol/L.

However, in tissues such as cartilage, diffusion limits the transport of $\mathrm{N_2}$: it takes time for it to diffuse and reach new equilibrium. Note that $\mathrm{N_2}$ is **not** produced nor consumed by cartilage or any living tissue. We call $n(x,t)$ the number of gas $\mathrm{N_2}$ molecules per m^3, and name D the diffusion coefficient of $\mathrm{N_2}$ in cartilage.



Q10. By continuity of $n(x)$ in 0 and L, determine the numerical values of $n(0)$ and $n(L)$.

Q11. Without any demonstration, express the differential equation that $n(x,t)$ follows here for $x \in [0, L]$.

We look for stationary solutions for $n(x,t)$. The plane at $x = \frac{L}{2}$ is Π^+ (a symmetry plane) for both $n(x,t)$ and $\vec{j}_{\mathrm{N_2}}$.

Q12. Demonstrate rigorously that $n(x,t)$ can be written as :

$$n(x,t) = n(0) + Ae^{-q_n^2 Dt}\sin(q_m x) \qquad \text{with } q_m = \frac{m\pi}{L} \text{ for } m \in \mathbb{N} \text{ and } A \text{ an unknown.}$$

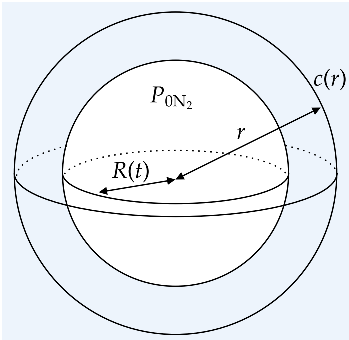
Q13. Determine the only value for m that makes sense physically, and justify why by representing graphically n(x,t) for different m.

At $t = 0$, the maximum of concentration in $\mathrm{N_2}$ within the cartilage $c_{\mathrm{N_2},\,eq,z=z_0} = 2\times 10^{-3}$ mol/L. The diffusion coefficient of $\mathrm{N_2}$ in cartilage is $D = 2 \times 10^{-9}$ m^2.s^{-1} and $L = 1$ cm.

Q14. Determine the numerical value of A, then define a characteristic time τ for the decay of this excess of concentration in N_2 within cartilage : how long should a diver typically take to reach the surface after diving at 30 meters ?

II.3 Growth of dinitrogen bubbles in living tissues

We study an isolated bubble of pressure P_{0N_2} and radius $R(t)$. The evolution of the radius $R(t)$ of the N_2 bubble is slow enough for the diffusion of N_2 in the liquid to be in steady-state. n is the number of dissolved molecules of N_2 per m^3 within the living tissue($\sim$ water) $n(r) \underset{r\to\infty}{\longrightarrow} n_\infty$. D is the diffusion coefficient of the gas in the liquid and V_n the molar volume of the gas, **supposed to be constant**.
To study $R(t)$ we neglect surface tension and give the following law :
- Henry's law : $n(R) = HP_{0N_2}$ with $H = 3,6 \times 10^{22}\ \mathrm{kg^{-1}.s^2.m^{-2}}$



In spherical coordinates for a scalar field $n(r)$ the gradient and Laplacian are written as such :

$$\overrightarrow{\text{grad}}\, c = \frac{\partial c}{\partial r}\overrightarrow{e_r} \qquad\qquad \Delta c = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rc)$$

Q15. Determine $n(r)$ using r,R, $n(R)$ and n_∞.

Q16. Using Fick's law, determine the volume variation rate $\dot{V}$ of the bubble per unit of time.

Q17. Show that along these assumptions the bubble's radius follows :

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{HDV_nP_{0N_2}}{R(t)}\left(\frac{n_\infty}{HP_{0N_2}} - 1\right)$$

Here $n_\infty = 1.1 \times 10^{27}$, $T = 310$ K, $D = 2 \times 10^{-9}\ \mathrm{m^2.s^{-1}}$, $R_{ig} = 8.3\mathrm{J.K^{-1}.mol^{-1}}$ and $P_{0N_2} = 0.8$ bar.

Q18. Demonstrate that the bubble will indeed grow, then define and determine the value of the duration Δt for it to grow to $R_0 = 1$ mm. Comment your result in regards of previous results for diffusion of dissolved N_2 in cartilage.

The diver suddenly reached the surface : the concentration of dissolved N_2 initially remained the one at equilibrium for $z = z_0$ (before the diffusion of the II.2 significantly occurs), but the partial pressure P_{0N_2} in N_2 dropped causing this rapid bubble growth.

Written exam n° 8, elements of correction:

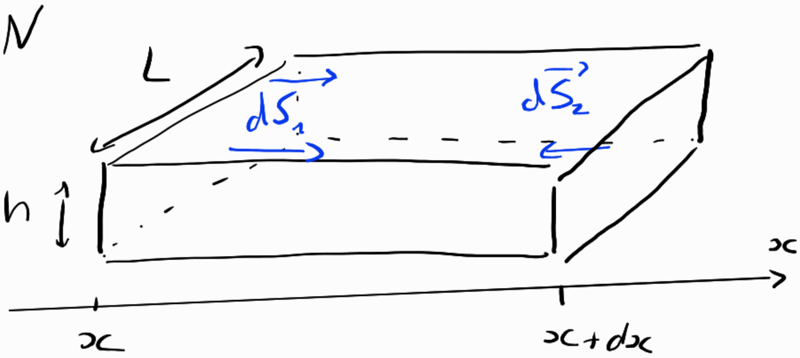
Q1. Fick's law states that $\vec{j_N} = -D\,\vec{\nabla} n$

Since $n = n(x,t)$, $\vec{\nabla} n = \frac{\partial n}{\partial x}\vec{e_x} + \frac{\partial n}{\partial y}\vec{e_y} + \frac{\partial n}{\partial z}\vec{e_z}$

Q2. Let us count d^2N

$d^2N = dN(t+dt) - dN(t)$

the variation of the number of perfume molecules between x and $x+dx$ between instants t and $t+dt$.



$\rightarrow d^2N = dN(t+dt) - dN(t) = hLdx\left[n\left(x+\frac{dx}{2}, t+dt\right) - n\left(x+\frac{dx}{2}, t\right)\right]$

$\rightarrow d^2N = dt \times \alpha \times Lhdx + dt \oiint \vec{j_N} \cdot d\vec{S}_{in}$

$$= \alpha Lhdxdt + \underbrace{dt\iint \underbrace{\vec{j_N}}_{\text{along } \pm\vec{e_x}} \cdot \underbrace{d\vec{S}_{in,lat}}_{\text{along } \vec{e_y} \text{ or } \vec{e_z}}}_{0} + dt\iint \vec{j_N}\left(x, t+\frac{dt}{2}\right)\cdot d\vec{S_1} + dt\iint \vec{j_N}\left(x+dx, t+\frac{dt}{2}\right)\cdot d\vec{S_2}$$

$$= Lhdt\left(\alpha dx + j_N\left(x, t+\frac{dt}{2}\right) - j_N\left(x+dx, t+\frac{dt}{2}\right)\right)$$

$$= Lhdxdt\left[\alpha - \frac{dj_N}{dx}\left(x, t+\frac{dt}{2}\right)\right]$$

Hence $\frac{\partial n}{\partial t}\left(x+\frac{dx}{2}, t\right) = \alpha - \frac{dj_N}{dx}\left(x, t+\frac{dt}{2}\right)$

since $dx \rightarrow 0$, $dt \rightarrow 0$
$$\frac{\left|\frac{dx}{2}\frac{\partial^2 n}{\partial x \partial t}(x,t)\right|}{\left|\frac{\partial n}{\partial t}(x,t)\right|} \ll 1 \quad \text{and} \quad \frac{\left|\frac{dt}{2}\frac{\partial^2 j_N}{\partial x \partial t}(x,t)\right|}{\left|\frac{\partial j_N}{\partial x}(x,t)\right|} \ll 1$$

thus $\frac{\partial n}{\partial t} + \frac{dj_N}{dx} = \alpha$

For $x \in \mathbb{R} \setminus [-x_0, x_0]$ $\alpha = 0$ hence $\frac{\partial n}{\partial t} + \frac{dj_N}{dx} = 0$

Q3. The diffusion equation is (using Fick's law + material balance):

$x \in [-x_0 ; x_0]$ | $x \in \mathbb{R} \setminus [-x_0 ; x_0]$

$$\frac{\partial n}{\partial t} - D \frac{\partial^2 n}{\partial x^2} = \alpha$$

$$\frac{\partial n}{\partial t} - D \frac{\partial^2 n}{\partial x^2} = 0$$

steady-state

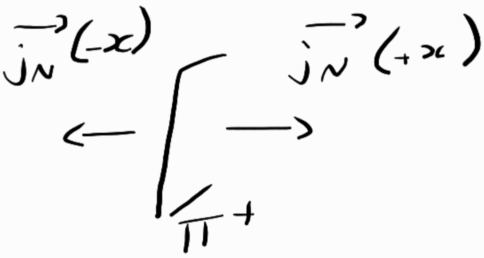
$$D \frac{d^2 n}{dx^2} = -\alpha$$

$$D \frac{d^2 n}{dx^2} = 0$$

$$n(x) = -\frac{\alpha x^2}{2D} + K_1 x + K_2$$

$$n(x) = K_3 x + K_4 \quad x < -x_0$$
$$n(x) = K_5 x + K_6 \quad x > x_0$$

Q4. By symmetry $\vec{j_N}(x=0,t) = \vec{0} \quad \forall t$ hence



$K_1 = 0$. $K_2 = n(0)$.

$$n(+x) = n(-x) \quad \forall x \in \mathbb{R} \Rightarrow \begin{cases} K_4 = K_6 \\ K_3 = -K_5 \end{cases}$$

Material balance in $x = \pm x_0$ implies

$j_N(x_0^-) = j_N(x_0^+)$ and $j_N(-x_0^-) = j_N(-x_0^+)$

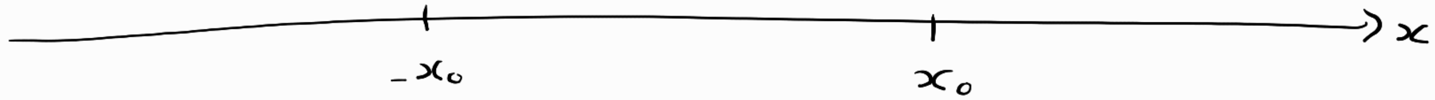
$$-\frac{\alpha x_0}{D} = K_5 = -K_3 \qquad K_3 = \frac{\alpha x_0}{D}$$

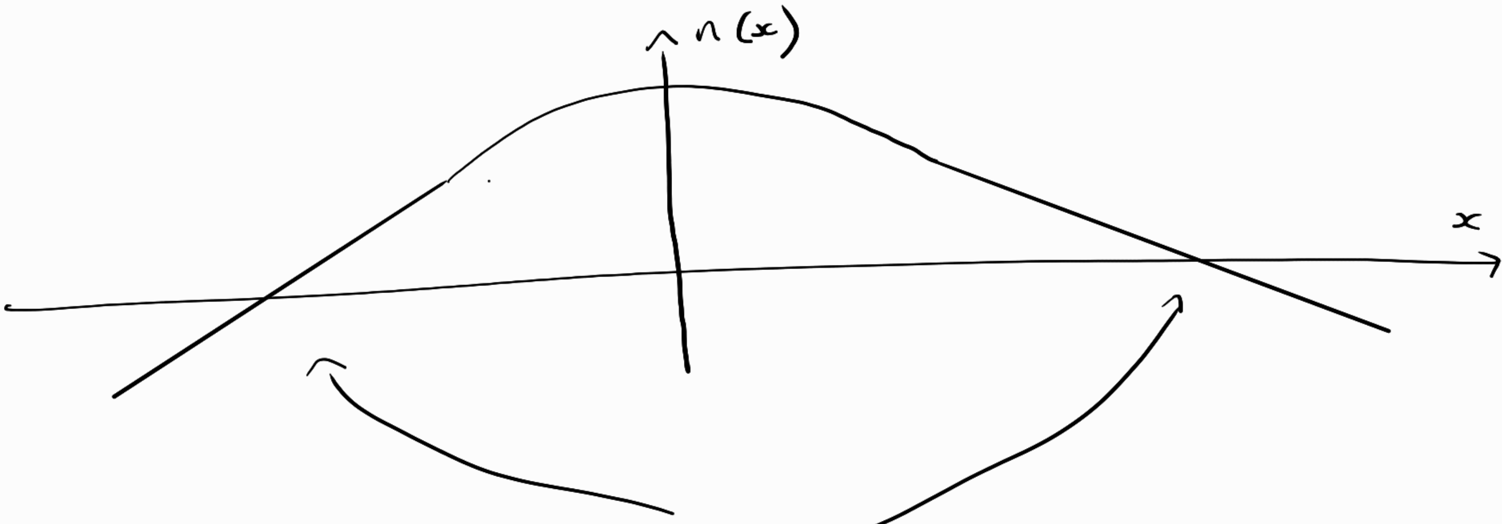
Without membrane n is continuous

hence
$$\begin{cases} n(0) - \frac{\alpha x_0^2}{2D} = -\frac{\alpha x_0^2}{D} + K_4 \\ n(0) - \frac{\alpha x_0^2}{2D} = -\frac{\alpha x_0^2}{D} + K_6 \end{cases}$$

$$K_4 = n(0) + \frac{\alpha x_0^2}{2D}$$

$n(0) + \frac{\alpha x_0^2}{2D} + \frac{\alpha x_0}{D} x$ $\quad$ $n(0) - \frac{\alpha}{2D} x^2$ $\quad$ $n(0) + \frac{\alpha x_0^2}{2D} - \frac{\alpha x_0}{D} x$





non-satisfying model, yet simple.

Q5. $\dot{N} = \iint \vec{j}_N(x_0) \cdot \vec{dS}$ with

$\vec{j}_N(x) = \alpha x$ hence $\dot{N} = \alpha x_0 L h$

Q6. 720 sprays empty the bottle that is $V_{spray} = \frac{5 \times 10^{-5}}{720} \simeq 7 \times 10^{-8}\ m^3$

$\underbrace{\text{bottle}}$ ↳ for a spray on $x \in [0; x_0]$ only and not $[-x_0; x_0]$

V_{spray} ↳ volume of perfume in 1 spray

with $N_{spray} = \frac{\mathcal{N}_A \rho_{perf} V_{spray}}{M}$ perfume molecules per spray

hence $\dot{N} = \frac{N_{spray}}{\Delta t}$ and $\alpha = \frac{\mathcal{N}_A \rho_{perf} V_{spray}}{M \Delta t\, x_0 L h}$

$$\alpha = \frac{6 \times 10^{23} \times 10^3 \times 7 \times 10^{-8}}{10^{-1} \times 5 \times 3600 \times 10^{-6}}$$

$$= \frac{7 \times 10^{18}}{3 \times 10^{-4}} \simeq 2{,}3 \times 10^{22}\ m^{-3} \cdot s^{-1}$$

Q7. $\tau \sim \frac{L^2}{D} \sim \frac{1}{3\times10^{-5}} = 3\times10^4 s \simeq 8h$ or so.

Convection obviously dominates at these scales as far as the flow is free.

Q8. Air contains a molar fraction of N_2 of around $0,8 = 80\%$.

Since $P_0 = 1$ bar, $P_{N_2}(z=0) = 0.8$ bar

and $c_{N_2,eq}(z=0) = 4,8\times10^{-4}$ mol/L

Q9. $P(z) = P_0 + \rho g z$ and $c_{N_2,eq}(z_0) = H P_{N_2}(z_0)$

and $\rho g z_0 = 3\times10^5$ Pa $= 3$ bar that is

$P(z_0) = 4$ bar and $P_{N_2}(z_0) = 3.2$ bar

hence $c_{N_2,eq}(z_0) = 19,2\times10^{-4}$ mol/L $\simeq 2\times10^{-3}$ mol/L

Q10. Initially at eq. $c_{N_2,eq}(z_0) = 2$ mmol/L within the blood. The new equilibrium requires $c_{N_2,eq}(0) = 0,5$ mmol/L, that is

$n = 7,5\times10^{-3}$ mol of N_2 are liberated.

That is a volume $V = \frac{nRT}{P} = \frac{7,5\times10^{-3}\times8\times310}{10^5}$

$V = 15\times10^{-5}\,m^3 = 0,15$ L, that is about the volume (in gas) of a glass of water.

Within a blood vessel of radius $R \simeq 3$ mm that is a length $L = \frac{V}{\pi R^2} = \frac{1,5\times10^{-4}}{3\times10^{-5}} = 5$ meters long of bubble ! needles to say that this is far from neglectable for the organism.

Q 11. The concentration of N_2 in blood is

$c_{N_2,eq} = 5\times10^{-4}\ mol/L$, that is $n(0) = n(L) = c_{N_2,eq}\times N_A$
$= 5\times10^{-1}\ mol/m^3$

$n(0) = n(L) = 3\times10^{23}\ m^{-3}$

Q 12. $\dfrac{\partial n}{\partial t} = D\dfrac{\partial^2 n}{\partial x^2}$

Q 13. stationnary solutions are such that

$n(x,t) = f(x)\times g(t)$ hence $\left[\dfrac{f'}{f}\right](t) = D\left[\dfrac{g''}{g}\right](x) = A$

with A a constant : $\begin{cases} f' = Af \\ g'' = \dfrac{A}{D}g \end{cases}$

$f = f_0 e^{At}$ and :

- $A>0$ $\quad g(x) = \alpha e^{-\sqrt{\frac{A}{D}}x} + \beta e^{\sqrt{\frac{A}{D}}x}$ will satisfy

  $g\left(\frac{L}{2}-x\right) = g\left(\frac{L}{2}+x\right)$ only for $(\alpha,\beta) = (0,0)$

- $A=0$ same.
- Hence $A<0$ $\quad g(x) = \alpha\cos\left(\sqrt{\frac{-A}{D}}x + \Psi\right)$

and $g\left(\frac{L}{2}-x\right) = g\left(\frac{L}{2}+x\right) \Rightarrow$

$$\cos\left(\sqrt{-\frac{A}{D}}\frac{L}{2}+\Psi\right)\cos x + \sin\left(\sqrt{-\frac{A}{D}}\frac{L}{2}+\Psi\right)\sin x = \cos\left(\sqrt{-\frac{A}{D}}\frac{L}{2}+\Psi\right)\cos x - \sin\left(\sqrt{-\frac{A}{D}}\frac{L}{2}+\Psi\right)\sin x$$

$$\Rightarrow \sin\left(\sqrt{-\frac{A}{D}}\frac{L}{2}+\Psi\right) = 0 \Rightarrow \Psi_k = k\pi - \sqrt{-\frac{A}{D}}\frac{L}{2}$$

$k\in\mathbb{N}$

Thus $g(x) = \alpha (-1)^k \cos\left(\sqrt{\frac{-A}{D}}\left(x - \frac{L}{2}\right)\right)$.

$A < 0$ thus $f(t) \underset{t \to +\infty}{\longrightarrow} 0$ yet $n(x,t) \underset{t\to+\infty}{\longrightarrow} n_0$, imposed by the surrounding blood. $n(x,t)$ satisfying boundary condition here is the superposition of n_0 the uniform profile in steady state + the stationnary function that one builds here: $n(x,t) = n_0 + f_0 e^{-At} \alpha (-1)^k \cos\left(\sqrt{\frac{-A}{D}}\left(x - \frac{L}{2}\right)\right)$

and $\begin{cases} n(0,t) = n(0) \\ n(L,t) = n(L) = n(0) \end{cases}$ impose:

$\cos\left(\sqrt{\frac{-A}{D}}\frac{L}{2}\right) = 0$ thus $\sqrt{\frac{-A}{D}}\frac{L}{2} = \frac{\pi}{2} + p\pi$

$$\sqrt{\frac{-A}{D}} = \frac{\pi}{L} + \frac{2p\pi}{L}$$

hence $g(x) = \alpha (-1)^k \cos\left(\left(\frac{\pi}{L} + \frac{2p\pi}{L}\right)\left(x - \frac{L}{2}\right)\right)$

$$= \alpha(-1)^k \cos\left(\frac{\pi x}{L} - \frac{\pi}{2} + 2\pi\frac{px}{L} - 2p\pi\right)$$

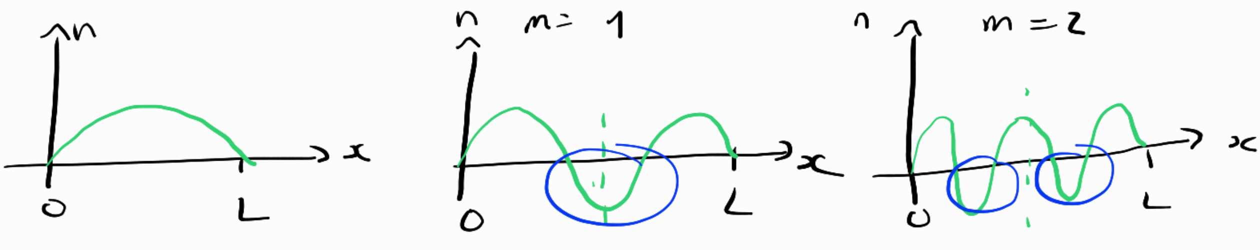
$$= \alpha(-1)^k \sin\left(\frac{\pi x}{L} + \frac{2\pi p x}{L}\right)$$

$$= \alpha(-1)^k \sin\left(\frac{\pi x}{L}(1+2p)\right)$$

call $m = p$, $q_m = \frac{(1+2m)\pi}{L}$ and $A = f_0 \alpha (-1)^k$

Q14. $n(x,t) \geq 0$ $\forall x$, and $\forall t$, hence, since $n(0)$ might be as small as 0, $A \sin(q_m x) \geq 0$ $\forall x \in [0;L]$.

$m = 0$



$\Rightarrow A \geq 0$ and $m = 0$ since $q_m x$ must be smaller than π $\forall x \in [0; L]$ yet

$$q_m L \leq \pi \Rightarrow \pi(1+2m) \leq \pi \Rightarrow m \leq 0$$

Q 15. $C_{N_2, eq, z=z_0} = 2 \text{ mol} . m^{-3}$

$$A + n(0) = \mathcal{N}_A \times C_{2, eq, z=z_0}$$

$$A = (12-3) \times 10^{23} = 9 \times 10^{23} \; m^{-3}$$

The exponential decay suggests to define $\tau = \dfrac{L^2}{\pi^2 D} = \dfrac{25 \times 10^{-6}}{9 \times 2 \times 10^{-9}} \simeq 1,4 \times 10^3 s$ $\quad \frac{70}{18} \simeq 4$

that is about 23 minutes : clearly it is necessary to be cautionous.

Q 16. Without source term the diffusion equation states for steady-state :

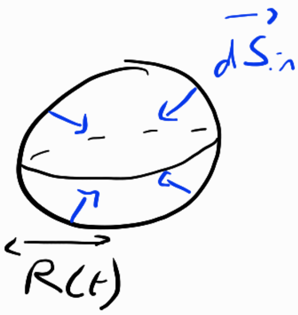
$n(r)$ and $\Delta n = 0$ hence $r^2 \dfrac{dn}{dr} = \alpha$

thus $n = -\dfrac{\alpha}{r} + \beta \xrightarrow[r \to +\infty]{} \beta = n_\infty$

and $n(R) = -\dfrac{\alpha}{R} + n_\infty$ hence

$$n(r) = \frac{n(R) - n_\infty}{r} R + n_\infty$$

Q 17. Using V_n, we can compute $\dot{V}$ using the quantity of N_2 diffusing within the bubble per unit of time, that we call $\dot{N}$



$$\dot{N} = \oiint \vec{j}(R)\cdot\vec{dS}_{in}$$

pointing inward, that is

$$\vec{dS}_{in} = -R^2 \sin\theta \, d\theta \, d\varphi \, \vec{e_r}$$

and $\vec{j} = D\,\dfrac{n(R)-n_\infty}{R}\,\vec{e_r}$ hence

$$\dot{N} = 4\pi D\,(n_\infty - n(R))\,R \quad \text{and} \quad \dot{V} = \frac{4\pi V_n D\,(n_\infty - n(R))\,R}{\mathcal{N}_A}$$

Q18. Geometrically

$$dt\,\dot{V} = V(t+dt) - V(t)$$
$$= \frac{4}{3}\pi\left[R^3(t+dt) - R^3(t)\right]$$
$$= 4\pi R^2(t) \times \frac{dR}{dt}\,dt$$

Hence

$$\frac{dR}{dt} = \frac{V_n D}{\mathcal{N}_A R}\,(n_\infty - \underbrace{n(R)}_{= H P_{ON_2},\ \text{Henry's law}})$$

Q19. $\dfrac{dR}{dt} > 0$ for $n_\infty > n(R)$

Here $n_\infty = 1.1\times10^{23}\ m^{-3}$

$n(R) = 8\times10^{4}\times 3.6\times10^{17} = 3\times10^{22}\ m^{-3}$

$n_\infty - n(R) > 0$ indeed.

$$R_0^2 = 2\frac{V_n D}{\mathcal{N}_A}\,(n_\infty - H P_{ON_2})\,\Delta t$$

$$\Delta t = \frac{R_0^2\,\mathcal{N}_A}{2 V_n D\,(n_\infty - H P_{ON_2})} = \frac{R_0^2\,\mathcal{N}_A\,P_{ON_2}}{2 R_g T\,(n_\infty - H P_{ON_2})}$$

$$\Delta t = \frac{10^{-4}\times 6\times10^{23}\times 8\times10^{4}}{2\times 8\times 300\times 8\times10^{22}} = \frac{6\times10^{23}}{5\times10^{25}} \simeq 10^{-2}\ s$$

That is 10 milliseconds only, clearly a lot faster than the tens of minutes required to get rid of it via blood flow redirecting the excess N_2 towards the lungs! In reality such growth neglects the $\left\{\begin{array}{l}\text{mechanical}\\ \text{diffusion}\end{array}\right.$ constraints of living tissue for which bubbles rather significantly grow in a few tens of seconds.